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Functional Thinking in Early Algebra: A Multi-Level Investigation of Elementary Mathematics Education in Indonesia

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ABSTRACT
Objective: This study aims to investigate functional thinking from three perspectives: (1) curriculum representation, (2) students' algebra difficulties during the elementary-secondary school transition, and (3) pre-service teachers' knowledge. **Method:** This study employs a qualitative approach through document analysis, as well as tests and interviews with students transitioning from elementary to secondary school and with pre-service teachers. **Results:** The results of this study indicate: (1) curriculum documents significantly underrepresent functional relationships compared to recursive patterns; (2) students demonstrate weak functional thinking during the transition to formal algebra; (3) pre-service teachers present limited content and pedagogical knowledge about functional thinking. These findings reveal deficiencies in functional thinking at all three levels, which implies the need to improve this aspect in elementary school mathematics instruction and in pre-service teacher education programs. **Novelty:** Functional thinking is the foundation for algebraic reasoning in elementary mathematics, which determines students' success at the next level. Despite its importance, little is known about functional thinking in Indonesian elementary mathematics education. This research fills this gap through a multi-level investigation to provide a comprehensive understanding.

INTRODUCTION

Mathematics learning in elementary education forms a foundation that supports students' success at higher levels. Algebraic thinking is not only necessary for mathematics learning in secondary education, but also strengthens students' reasoning at earlier levels. Algebraic thinking is essential for students because this ability serves as a crucial foundation for mathematical competence and reasoning literacy in modern life (Mason, 2017a; Perry, 2022), as well as enhancing overall cognitive development (Mason, 2017b). Research also shows that algebraic thinking skills play a role in improving the understanding and application of STEM concepts (Odondi & Aje, 2025). For more than two decades, a paradigm shift in mathematics education has emerged, indicating that algebraic reasoning—encompassing generalization, representation, and the analysis of relationships among quantities—should begin in elemantary school (M. Blanton et al., 2023; M. L. Blanton & Kaput, 2011; Kaput, 2008; NCTM, 2000). Early exposure to algebraic reasoning skills such as generalization and implicit reasoning is needed by students in the transition from arithmetic to algebra and can bridge the gap across different educational levels (Dekker & Dolk, 2010; Stylianou, 2024).

The study of early algebra learning can be viewed multidimensionally. Kaput (2008) outlines two core aspects of early algebra learning: (1) making and stating generalizations in increasingly formal and conventional systems, and (2) working with symbols in organized symbol systems and established syntax. Furthermore, Kaput (2008) also explains that these two core aspects of algebra are found in three content

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areas: (1) algebra as the study of structures and systems abstracted from computation and relations, including those arising from arithmetic and quantitative reasoning, (2) algebra as the study of functions, relations, and joint variation, and (3) algebra as the application of modeling language clusters both within and outside mathematics. Similarly, Kieran (2022) categorizes three dimensions of early algebraic thinking as Analytic Thinking, Structural Thinking, and Functional Thinking. Analytic and structural thinking are rooted in the generalized-arithmetic perspective, where numbers are treated as basic mathematical objects. Meanwhile, functional thinking is based on the perspective of functions as fundamental mathematical objects. Functions were initially viewed as a small part of algebra, but since the new mathematics curriculum movement of the 1980s, functions have become a full component of school algebra (Kieran, 2022). Following this paradigm shift in the mathematics curriculum, functional thinking has emerged as an aspect widely studied by researchers in an effort to holistically improve students' algebraic and mathematical abilities. Blanton & Kaput (2011) conceptualize functional thinking as the process of constructing and generalizing patterns and relationships using a variety of linguistic and representational tools, and treating these generalized relationships as distinct mathematical objects.

Patterns are a natural and intuitive starting point for entering algebraic thinking. Radford & Peirce (2006) state that pattern recognition can help students see regularities, realize structures, and gradually begin to use symbolic representations. Mason (2005) also emphasizes that patterns can serve as the foundation for building awareness of relationships, changes, and structures, which are the starting point for algebraic awareness. The aforementioned branches of algebraic content demonstrate that pattern recognition is not only necessary for abstracting a sequence of numbers but also highlights the importance of recognizing patterns of relationships between quantities or functional relationships. Much of the discussion has focused on how and where functional thinking is incorporated into pattern recognition learning.

Blanton & Kaput (2011) argue that functional thinking is a pathway to developing algebraic thinking in elementary grades. Pattern recognition can begin with recursive thinking, for example, by determining the next term based on the previous term (e.g., each term is three times the previous term), then progressing to explicit thinking, such as stating general relationships using symbols (e.g., the nth term = 3n -1). The transition from recursive to implicit forms is a key component in the development of functional thinking (Ratnasari et al., 2023). The concept of functional thinking formulated by Blanton & Kaput (2011) is based on Smith's (2008) framework of three modes for analyzing patterns and relationships: recursive patterning, covariational thinking, and correspondence relationships. Recursive patterning is related to recognizing patterns in a sequence of values, while covariational thinking and correspondence relationships are related to recognizing patterns of relationships between two quantities. Thus, the domain of functional thinking is the ability of correspondence and covariational thinking.

Essentially, correspondence thinking is the ability to coordinate input and output values (e.g., when x = 3, y = 7). Meanwhile, covariational reasoning is concerned with understanding how two quantities change simultaneously (Thompson & Carlson, 2017). If correspondence thinking asks 'what is the value?', then covariational reasoning asks 'how does it change?' Covariational reasoning is seen as a thinking skill that is essential for reasoning advanced mathematical concepts, but on the other hand, it also presents

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many challenges even for students at the high school and university levels, where it requires students' ability to construct multiplicative objects (Thompson, 2011). Nevertheless, empirical evidence shows that elementary school children can develop an understanding of algebra that includes generalized arithmetic and functional thinking (Brizuela et al., 2015; Chimoni et al., 2018).

International studies on the importance of developing functional thinking at the elementary school level have encouraged efforts to emphasize this aspect in the curriculum (Ding et al., 2023; Hemmi et al., 2021; Perry, 2022). Based on the results of the 2022 Programme for International Student Assessment (PISA), Indonesian students have not yet achieved satisfactory mathematics skills (OECD, 2022), prompting academics and policymakers to transform mathematics learning in Indonesia through the Independent Curriculum (Kurikulum Merdeka). However, efforts to improve the quality of mathematics education still require more effort. For example, previous research shows that many students still experience difficulties in solving change and relationship problems (Khusnah et al., 2022; Yusepa et al., 2025), which may be related to how the Indonesian curriculum provides space for the development of students' functional thinking. A comprehensive study is needed to examine the extent to which opportunities for developing functional thinking are truly represented in the Indonesian mathematics curriculum. However, this research remains very limited. An analysis of functional thinking aspects in the curriculum was found in a study by Utami et al. (2023), but was limited to an analysis of textbooks at the junior high school level.

The transition from elementary school to secondary school is often a challenging phase for students, particularly the shift from arithmetic to algebraic thinking (Apsari et al., 2020). Previous research indicates that functional thinking remains a challenging area for students (Utami, Prabawanto, & Suryadi, 2023). One specific difficulty that continues to emerge is the ability to understand covariation, namely how two quantities change simultaneously in a pattern or function. Wilkie's (2020) findings revealed that many students are unable to consistently link changes in two variables when asked to identify or establish functional relationships. In Syawahid et al.'s (2020) study on functional thinking skills in elementary school students in Indonesia, covariational thinking was also not found. This contradicts several studies in international contexts that show that it is possible to develop covariational reasoning as early as elementary school (Andre et al., 2022; Chimoni et al., 2018; Deng et al., 2025; Paoletti et al., 2024; Sterner, 2024). Although findings on the difficulties in developing functional thinking have been widely discussed in international contexts, research that deeply examines how Indonesian students experience similar difficulties is still very limited. Meanwhile, *Kurikulum Merdeka* in Indonesia generally stipulates that the field of algebra studies at the elementary level includes the progression from non-formal algebra to formal algebra, including relations and patterns (BSKAP, 2025).

In addition to curriculum documents, teacher preparedness is also a crucial factor in ensuring that functional thinking develops systematically in the classroom. Teachers' content knowledge and pedagogy, particularly Pedagogical Content Knowledge (PCK) (Ball et al., 2008; Hill et al., 2008), play a crucial role in determining how mathematical concepts are taught and understood by students. However, many teachers have limited pedagogical knowledge of functional thinking content for early algebra instruction (Pincheira & Alsina, 2021). A study by Öztürk et al. (2025) found a lack of knowledge

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among prospectiv teachers in guiding elementary-level algebraic thinking, including functional relationships. However, they also provided evidence that prospective teachers' abilities can be developed through interventions. This situation highlights the urgent need for more comprehensive knowledge of functional thinking, especially for pre-service teachers who have not had sufficient experience in teaching early algebra.

The Indonesian educational context provides a suitable case for investigation. Recent curriculum reforms, particularly Kurikulum Merdeka 2022, have emphasized mathematical reasoning and 21st-century skills. However, the extent to which functional thinking is included in the curriculum as a distinct component of reasoning has not been explicitly and clearly discussed. As a highly diverse education system undergoing reform, Indonesia can provide valuable insights for international comparative research on early algebra learning.

This study aims to examine functional thinking in Indonesian elementary mathematics education through three complementary levels: curriculum, students, and pre-service teachers. The research questions include:

RQ1: To what extent are opportunities for functional thinking represented in Indonesian elementary school mathematics curriculum documents?

RQ2: What difficulties in functional thinking do students exhibit during the transition from elementary to secondary school, and how do they perform on formalization tasks for early algebra?

RQ3: What is the functional thinking knowledge of pre-service mathematics teachers, specifically related to content knowledge and pedagogical content knowledge regarding teaching early algebra?

This research is expected to contribute to the expansion of theory on early algebra learning with comprehensive empirical evidence based on a multi-level study in Indonesia. The results can practically inform curriculum, teaching, and teacher education. From a more global perspective, the research findings can be utilized not only in the context of education in Indonesia but also provide a comparative perspective for the global community. The findings also provide evidence that can be used as consideration in educational reform.

RESEARCH METHOD

This research employs a qualitative approach with a multi-level case study design examining functional thinking in Indonesian elementary mathematics education through three levels of analysis: curriculum, students, and pre-service teachers. The documents analyzed are those applied at the national level, while data collection involving students and pre-service teachers was conducted in East Java Province. The qualitative approach was chosen because it enables an in-depth exploration of the complex phenomena related to functional thinking within the context of Indonesian mathematics education, and provides a holistic understanding of how functional thinking is represented in elementary school mathematics curriculum documents, how it is understood by students during the transition to secondary education, and how pre-service teachers comprehend the content and pedagogy of early algebra instruction that encompasses functional thinking.

The research subjects for RQ1 consist of Indonesian curriculum documents, including the Mathematics Learning Outcomes (CP) Documents obtained from the official portal, the Mathematics Subject Guidebook, and the textbooks published by the

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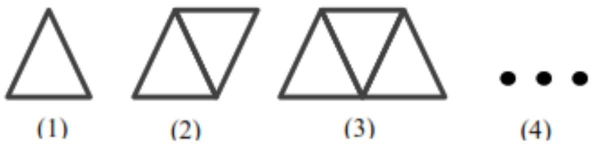
Indonesian Ministry of Elementary and Secondary Education. The researcher collected data from textbooks that potentially contain elements of functional thinking, specifically chapters or sections addressing patterns and functional relationships. The researcher then described the occurrence of each topic related to functional thinking, including the grade level and phase of instruction.

Meanwhile, the research subjects for RQ2 are 35 seventh-grade junior secondary school students who have completed the learning content in the early algebra elements of Phase D, namely patterns and algebraic expressions. The selection of seventh-grade students aims to capture the effects of learning in elementary school and assess their readiness for the transition from arithmetic and pattern-based thinking to formal algebra. The data obtained from these subjects serve as a foundation for providing evidence of the extent to which students demonstrate functional thinking skills. The research site for this data collection was a private junior secondary school in Jombang Regency, East Java. To obtain data on students' functional thinking and algebraic understanding, the researcher employed a written test instrument requiring students to model word problems into algebraic forms, as presented in Figure 1. After the test was administered to students, semi-structured interviews were conducted to clarify students' reasoning in solving the problems. These algebra problems are important for demonstrating students' ability to generalize patterns using variables, which indicates their success in transitioning to formal algebra.

MATCHSTICK AND TRIANGLES PROBLEM

1. Observe the following matchstick arrangement pattern!



Siti wants to connect matchsticks of equal length to form several adjacent triangles. The first triangle requires 3 matchsticks, and each subsequent triangle requires an additional 2 matchsticks. Based on this statement:
   a. Select and write a variable to represent the number of triangles created.
   b. Write an algebraic expression that represents the number of matchsticks needed to form several adjacent triangles.
   c. How many matchsticks are needed to form 10 adjacent triangles?
   d. If each matchstick is 3 cm long, determine the total length of matchsticks needed to form 10 adjacent triangles.

HOURLY WAGE PROBLEM

2. Budi earns Rp. 50,000 per hour working at a bookstore.
   a. Select and write a variable to represent the number of hours Budi works per week.
   b. Write an algebraic expression that represents Budi's total weekly earnings.
   c. How much will Budi earn if he works 20 hours this week?

Figure 1. Early algebra problems

This algebra problem instrument consists of matchstick problem and hourly wage problem, which are commonly used in textbooks and teaching materials as main activities in algebra learning. The key to the matchstick problem is recognizing and extending patterns and generalizing linear functional relationships (Gierdien, 2012). Meanwhile, in the hourly wage problem, students are asked to describe the functional relationship between dependent and independent variables and express it in a general

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equation (Swafford & Langrall, 2000). The test instrument underwent expert validation by two mathematics education lecturers specializing in school algebra instruction. Expert validation assesses the accuracy of the presentation and the translation of the adapted problems within the Indonesian student context, such as adjusting the language and changing the dollar currency to rupiah. The expected answers to the problems are presented in Table 1. This test required students to: (a) identify variables and the relationships among them, and (b) express contextual situations using formal algebraic representations.

Table 1. Expected Student Responses to the Problems

Nu	Expected Responses
1a	Let the number of triangles be represented by the variable *n*.
1b	3 + (n-1) 2 = 3+2(n-1)= 3+2n-2 = 2n+1 as the algebraic expression representing the number of sticks needed to form *n* triangles.
1c	To form 10 triangles, 21 sticks are needed.
1d	If each stick is 3 cm long, then the total length of sticks for 10 triangles is 21 × 3 = 63 cm.
2a	Let Budi's weekly working hours be represented by the variable *x*.
2b	Since Budi earns 50,000 rupiah per hour, his weekly wage is represented as 50,000*x* rupiah.
2c	If Budi works 20 hours in a week, then his earnings are 50,000 × 20 = 1,000,000 rupiah.

Furthermore, to address RQ3, data on pre-service teachers' content and pedagogical knowledge were obtained from an assignment requiring the design of problems for early algebra instruction in Grade VII. This test was administered to 113 pre-service teachers, from which 6 were selected for interviews or reflective discussions to explore the pedagogical reasoning underlying their problem selection. The selection of the 6 interview subjects was based on maximum variation considerations regarding the characteristics of the response types demonstrated. These responses were identified based on the uses of variables contained in the problems proposed by the pre-service teachers according to the 3UV (Three Uses of Variable) model proposed by Ursini & Trigueros (2001), which include unknown number, general number, and functional relationship. The use of variables as a functional relationship served as an indicator of pre-service teachers' attention to the development of students' functional thinking. Both the task instrument and interview protocol underwent validation by two experts in mathematics and mathematics education.

The analysis began with initial coding of all data to identify segments related to functional thinking, including patterns and generalization. Thematic coding was conducted during the data analysis process based on Blanton and Kaput's framework (2011), namely recursive patterning, covariational thinking, and correspondence relationships. To ensure the quality of this qualitative study, several strategies were employed. Data triangulation using three sources (curriculum, students, and pre-service teachers) was utilized to provide a comprehensive understanding of functional thinking within the Indonesian mathematics education system. In addition, the researcher engaged in discussions with fellow researchers and mathematics education experts in interpreting the data and documented the entire research process systematically.

The researcher adhered to research ethics principles, particularly those concerning human subjects. Efforts to comply with these ethical standards were carried out by obtaining permission from the relevant educational institution before conducting the research, securing informed consent from the participants, ensuring the confidentiality

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and anonymity of research subjects, and using the research data solely for academic purposes.

RESULTS AND DISCUSSION

Results

The following section presents the findings of the data analysis, which consist of an analysis of the representation of *functional thinking* in early algebra based on the current curriculum documents, students' test results during the transition to formal algebra, and pre-service mathematics teachers' knowledge of functional thinking in the context of designing early algebra instruction.

Functional Thinking in the Curriculum

The documents analyzed in this study include the Learning Outcomes for Phase A through Phase D within the algebra element, supplemented by the Mathematics Subject Guide and relevant textbooks. The analysis of the mathematics textbooks was limited to chapters that explicitly address pattern topics and early algebra. Pattern instruction was identified in Phase A in the chapter *"Patterns – Object Positions and Patterns in Everyday Life"* for Grade II, and in Phase B in the chapter *"Visual Patterns and Number Patterns"* for Grade IV. In Phase C, the algebra element appeared in the chapter *"Ratio"* for Grade VI. Phase D contains a substantial algebra component, including the introduction of algebraic expressions, relations, functions, equations, and inequalities. However, as this study focuses on early algebra, the analysis was restricted to the transition to formal algebra found in the chapter *"Algebraic Expressions"* for Grade VII.

Based on the analysis of the three types of documents, a relatively strong and explicit form of *recursive patterning* is found in the algebraic elements of Phase A, particularly in the formulation of the Learning Outcomes through the phrase "…recognizing, imitating, and extending patterns…". For example, in Phase A, students are asked to identify what repeats in patterns across four types of patterns: shape, color, movement, and sound. Concrete examples of patterns are also provided, such as the explicit statement "the pattern is boy, girl, boy, girl, boy, girl." Meanwhile, *functional relationships* do not yet appear at this phase. Although patterns are addressed, students are only asked to extend the next term and are not required to determine the nth-term.

In Phase B, students begin to be introduced to growing patterns that "increase or decrease." Several examples of growth rules for concrete objects are presented recursively, such as "increasing by 3 at each step." This direction of growth actually has the potential to serve as an aspect of *covariational thinking* if articulated through questions such as "How do the numbers change as the position changes?" However, this is unfortunately not expressed in either the Teacher's Guide or the textbook. *Correspondence thinking* is also not strongly present at this phase. The only examples that point toward correspondence thinking are specific questions asking for the value of a particular term, such as "How many triangles are there in the 10th position?" or "In the picture, there are only three groups of oranges. Do you know what the seventh group would look like?"

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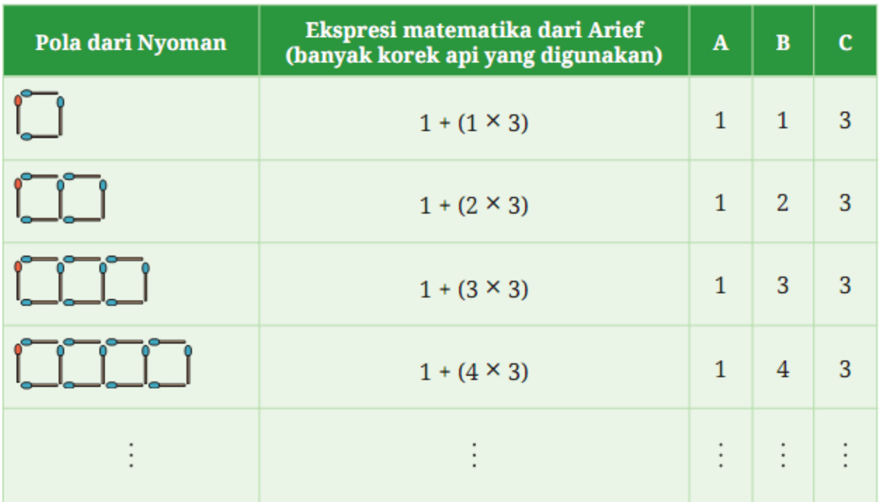
In Phase C, an introduction to patterns involving changes in quantity through multiplication and addition begins to appear; however, the relationships between these quantities are not explicitly stated. The pattern examples at this phase are more complex, involving rules of geometric number patterns such as 3, 9, 27, 81 (multiplying by 3) and 200, 50, 12.5 (dividing by 4). Early potential for covariational thinking is fairly strong in Phase C within the topic of ratio and proportion, yet there are no questions addressing "relationships" or "correlations." The initial seeds of covariational reasoning appear in examples of proportional reasoning in rectangles: when the width changes from 2 to 6 (multiplied by 3), the length also changes from 5 to 15 (also multiplied by 3). However, these examples remain limited to discrete relationships and rely on a few specific cases.

Similarly, correspondence thinking is not explicitly articulated. Although this potential could be developed from proportion, there is no discussion of functional relationships between position and value. For instance, a ratio table of land area and seedlings could be further developed into a functional relationship between seedlings and land area, as in the example below:

Land area (m²)	**Seedlings**
10	12
25	30
Number of seedlings	**Price**
3	Rp37,500
1	Rp12,500 (unit price)

The generalization of patterns through determining an nth-term formula emerges more prominently in Phase D. The introduction of patterns is reinforced in this phase, guiding students toward pattern generalization through observations of visual representations and tables. For example, students are asked to observe the increase of 3 matchsticks for every additional square, and a table representation is provided that breaks down the components of the pattern into columns for fixed values and changing values, as illustrated in Figure 2.



Pola dari Nyoman	Ekspresi matematika dari Arief (banyak korek api yang digunakan)	A	B	C
	1 + (1 × 3)	1	1	3
	1 + (2 × 3)	1	2	3
	1 + (3 × 3)	1	3	3
	1 + (4 × 3)	1	4	3
⋮	⋮	⋮	⋮	⋮

Figure 2. Pattern representations in the Grade VII textbook used to introduce the concept of variables and formal algebraic expressions

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Pattern generalization is extended to the point where students are guided to derive an algebraic expression with a variable as a general number, as shown in Figure 3. This section can be considered a transition from arithmetic thinking to algebraic thinking. This transition involves correspondence relationships more explicitly, whereas covariational thinking within these examples is not explored.

menggunakan sebuah huruf. Dalam hal ini, ayo kita gunakan huruf n, maka ekspresi matematikanya akan menjadi:

$$1 + (\_\_\_ \times 3) = 1 + 3\_\_\_$$

Ekspresi matematika yang menggunakan huruf ini disebut sebagai **bentuk aljabar**. Di dalam istilah formal matematika, kita menyebut huruf n tersebut sebagai **variabel**. Kalian juga dapat menggunakan simbol untuk menyatakan suatu variabel.

Translation:
In this case, let us use the letter n, so the mathematical expression becomes:

1+(__×3) = 1 + 3__1

A mathematical expression that uses a letter is called an **algebraic expression**. In formal mathematics, such a letter is referred to as a variable. You may also use a symbol to represent a **variable**.

Figure 3. The transition from recursive patterning to correspondence thinking in the Grade VII textbook for introducing variables

In Phase D, there is a significant and noticeable increase in the emphasis on *correspondence thinking*, which focuses on input–output relationships when introducing the concepts of relations and functions. This section concentrates on function evaluation (input–output), rather than on processes of change. *Covariational thinking* does not appear at the beginning of Phase D, although some implicit potential is present in several parts following the introduction of formal algebraic forms, such as linear and nonlinear graphs. Several statements discussing ratios also indicate a direction toward thinking about the simultaneous relationship between two quantities, such as, "Banyu can cover 1 km in 30 minutes" and "If one quantity in a ratio is multiplied by a certain number, then the other quantity is also multiplied by the same number,"; however, the general covariational relationship is not explored further. A summary of the functional thinking components in the curriculum document is presented in Table 2.

Table 2. Representation of Functional Thinking in Curriculum Documents

Phase	Recursive Patterning	Covariational	Correspondence
A (grade 1-2)	Strong and explicit, through systematic introduction of repeating patterns with four types of patterns: shape, color, movement, and sound	Not found	Not found
B (grade 3-4)	Strong and explicit, including statements on the direction of growth ("expanding and shrinking patterns")	Implicit potential in examples of increasing and decreasing number patterns, but the covariation aspect is not articulated	It is not found in the Learning Outcomes formulation or the Teacher's Guide; however, there is a limited example in the textbook that serves as an initial introduction to a correspondence relationship.
C (grade 5-6)	Mentions additive and multiplicative pattern changes, although pattern	Relatively strong implicitly through proportional	Contains hidden potential when proportions are viewed as correspondence, but no

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Phase	Recursive Patterning	Covariational	Correspondence
	growth is not the main focus	reasoning as an early form of covariational thinking	indication that this was the intended focus
early D (grade 7)	Very strong and explicit, starting from observing patterns to making generalizations and determining the nth term	No discussion of simultaneously changing relationships	Very strong, with an emphasis on creating generalizations to determine the value of the nth term as a pathway to formal algebraic expressions

Based on the analysis of the documents from Phase A to Phase D within the early algebra element, *recursive patterning* is stated explicitly and remains dominant across phases, with increasing levels of pattern complexity. Meanwhile, *functional thinking* is relatively weak in Phases A to C, although some implicit potential found in several examples could be further developed through reflective questioning. Early indications of *correspondence thinking* become more evident in the discussions on ratio, yet the connection to other components is not clearly established. Upon entering Phase D, there is a significant increase in the use of a *correspondence relationship* approach, whereas *covariational thinking* remains underdeveloped.

A crucial part of Phase D lies in the transition from pattern recognition to formal algebraic forms. In this section, the shift from *recursive patterning* to *functional thinking* begins to appear more clearly, although it remains limited to a correspondence-based approach. In this phase, students are introduced to pattern generalization for determining the nth term. The Phase D documents also begin to incorporate multiple representations of patterns—visual diagrams, tables, and mathematical statements—which, in fact, hold potential for deepening *covariational thinking* by comparing patterns from two sequences that share a functional relationship.

Students' Functional Thinking in the Transition to Algebra

Based on the test results, it was found that all 35 students were unable to answer the early algebra problems completely and correctly. This outcome indicates that the students had not yet succeeded in understanding the meaning of variables and formal algebraic forms, even though they had received instruction on these topics. While some of them were able to observe visual pattern diagrams, they still faced challenges in interpreting variable symbols within the generalized pattern forms they observed. In Table 3, we present three examples of student responses—Asa, Lily, and Nafa (pseudonyms)—that represent the typical answers found in this study.

Several issues identified from these findings indicate that some students do not understand the term *variable*, as evidenced by their tendency to write a number when asked to state a variable. Other students used conventional variable symbols such as "x" and drew on prior experiences with algebraic expressions, yet they used these symbols without meaning. At times, students interpreted variables as symbols representing objects (for example, *l* representing the first letter of *lidi*—the Indonesian word for "stick"—and *m* representing the first letter of *minggu*—the Indonesian word for "week") and were unable to use them to express a general number or a functional relationship.

Most student responses demonstrated a tendency to manipulate and operate on the given numbers in the problem randomly as a reaction to their confusion, a pattern consistently seen in Item 1d. Some students demonstrated arithmetic computation

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skills, even though they were unable to represent the given situation in algebraic form. This phenomenon is illustrated by Asa's response to Item 2c, in which they produced the correct numerical answer without using any algebraic expression.

Table 3. Typical Student Responses Showing Difficulties in Early Algebra.

Item	Asa's Answer	Lily's Answer	Nafa's Answer	Interpretation
1a	R	X	L	Nafa write L for "*Lidi*"
1b	7 x 1 = 7	3x + 2 = 10	2a + 3	Students typically use the numbers provided in the question, such as 2, 3, and 10, but do not apply them correctly.
1c	20	10x + 3 = 8	21a + 3	
1d	30 lidi	10x + 3 = 30 cm	30 cm	
2a	110 jam	A	M	Asa write number instead of variable symbol. Nafa write M for "*Minggu*"
2b	50.000	50.000a + 20	350.000M	Students typically use the numbers given in the problem, such as 50,000 and 20, but do not apply them correctly. Meanwhile, Asa was able to correctly perform the arithmetic calculations in Question 2d without using algebraic expressions.
2c	1.000.000	50a + 20 = 70.000 rupiah	1.000M	

These results suggest that strategies promoted in the textbook for generalizing patterns to derive algebraic expressions remain quite challenging for most students. Those who are accustomed to viewing a single pattern struggle to shift toward thinking about patterns involving relationships between two quantities (for instance, the relationship between position and value). This indicates that *functional thinking* should be nurtured from an early stage, before students enter lower secondary school.

Pre-service Mathematics Teachers' Knowledge of Functional Thinking in Designing Early Algebra Instruction

Based on the assignment in which pre-service mathematics teachers were asked to design problems for early algebra instruction, the submitted tasks were predominantly focused on the use of variables as *specific unknown numbers* and on arithmetic problems that did not involve variables. Problems that incorporated functional relationships appeared only twice across all submissions. Table 4 shows the number of early algebra word problems classified by categories of variable use.

Table 4. The early algebra word problems classified by categories of variable use

The Use of Variable in Word Problem		N
a.	Specific unknown number	80
b.	Functional relationship	2
c.	No variable (constant)	20
d.	Unclear	4

An example of a problem that uses variables in a word problem to express a functional relationship is shown in the following item. The use of variables in this problem reflects a dynamic view—representing quantities that change simultaneously, namely the amount of money saved and the number of months.

> *Mr. Anang is a teacher.* Every *month, he saves Rp150,000 in a savings box. The box already contains Rp2,500,000. What is the algebraic expression for the amount of money in Mr. Anang's savings box each month?*

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Based on interviews with six pre-service teachers representing each type of variable use in the submitted problems, it was found that their knowledge of variables was still dominated by a static understanding of variables as objects or as specific unknown numbers, as shown in Table 5. Only one participant demonstrated functional thinking through a problem that involved the use of variables as a functional relationship.

Table 5. Pre-service teachers' conceptions of variables

Knowledge abaout variable	The Number of Subject	Refference	Subject
Dynamic/Functional Thinking	1	1	S1
Static	5	19	S2, S3, S4, S5, S6
1) Object	3	12	S3, S5, S6
2) *Specific Unknown Number*	4	7	S2, S3, S4, S6

Discussion

The findings of this study, viewed across the three dimensions, demonstrate consistency and provide a comprehensive understanding of the presence of functional thinking in early algebra within Indonesia's school mathematics curriculum. The curriculum content analysis indicates that mathematics instruction at the elementary level places stronger emphasis on recursive patterning, while aspects of functional thinking remain underdeveloped. In comparison, elementary school textbooks in other countries, such as Japan, South Korea, and China, place explicit emphasis on covariational reasoning and corresponding relationships (Ding et al., 2023; Gantt et al., 2023; Kim & Kim, 2025; Pang & Sunwoo, 2022). Yet all branches of algebraic thinking are interconnected in shaping students' overall level of algebraic reasoning (Chimoni et al., 2018). As part of ongoing international discussions, functional thinking needs to be introduced early in primary school as a foundation for algebraic success at the secondary level (M. Blanton et al., 2023; Knuth et al., 2016).

Although elements of functional thinking appear in several sections of the curriculum documents, their occurrences lack coherence across phases. The connection between recursive patterning and functional thinking becomes noticeable only in Phase D (Grade VII), and even then, the focus is primarily on correspondence thinking, with limited emphasis on covariational thinking. However, students' understanding of covariation plays an essential role in helping them construct and interpret correspondence rules effectively (Ellis et al., 2016).

Given the design of learning outcomes and textbook content in earlier phases, the transition from arithmetic pattern generalization to variable introduction and formal algebraic expressions appears less than smooth. This effect is confirmed by the significant difficulties revealed in this study as students enter the transition toward formal algebraic reasoning. While students are able to recognize patterns and engage in arithmetic reasoning, they struggle to understand the meaning of variables derived from generalized positional terms of the nth number pattern. Throughout primary mathematics education, students are primarily exposed to single-sequence patterns, yet have limited experience in abstracting relationships between two simultaneously varying quantities. This transition is indeed challenging, as it requires various forms of reasoning to shift from recursive relationships in patterns toward identifying co-variational and correspondence relationships (Chimoni et al., 2018).

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The findings also align with the characteristics of the problems constructed by the pre-service mathematics teachers who participated in this study, as well as with their understanding of variables. Slabý et al. (2023) found that teachers tend to be stronger in their knowledge of "functions as formulas," while being weaker in aspects of functional thinking. The dominance of early algebra problems involving variables as unknown numbers may also be linked to the reference materials used by the pre-service teachers, most of which come from school mathematics textbooks. In contrast, teachers in East Asian countries such as South Korea generally have a deep understanding of specific mathematical topics such as covariational reasoning (Yang et al., 2025), which aligns with the content of the country's elementary school textbooks that reinforce all aspects of functional thinking. Teachers' decisions in selecting tasks typically emerge as a critical issue because they tend to align with the cognitive demand of textbook problems (Son & Kim, 2015), while the criteria for textbook selection influence teacher performance (Shalgimbekova et al., 2024).

The limited knowledge of pre-service teachers regarding variables—and the misconceptions observed—suggests a static view of variables. They struggle to see the dynamic nature of quantitative values that change continuously. Yet functional relationships in mathematics often require dynamic interpretation, such as understanding rates of change (Rolfes et al., 2020). Their thinking may also reflect the effects of their prior learning experiences during schooling, which may not have emphasized dynamic perspectives in functional thinking, especially covariational reasoning. Prior learning experiences of pre-service teachers are known to strongly influence their beliefs about teaching mathematics (Lo, 2021, 2023).

The pedagogical content knowledge limitations revealed in this study highlight the importance of professional development programs for primary school teachers that specifically address these weaknesses. Wilkie (2016) recommends sustained and structured professional development that focuses on multiple representations, developing teachers' understanding of students' learning progressions in functional thinking, and emphasizing relational rather than merely procedural understanding. Pre-service teachers need training to explore patterns as relationships between variables, rather than simply as procedures for calculating subsequent terms (Mcauliffe & Vermeulen, 2018; Wilkie, 2014). Strengthening both the curriculum and teacher education dimensions will create meaningful systemic improvement in early algebra learning in Indonesian schools, thereby enhancing students' algebraic thinking skills.

Several instructional strategies proposed in prior research for teaching functional thinking include the use of multiple representations (Hansen & Nerida F, 2015; Pittalis et al., 2025; Wei et al., 2025), the use of digital simulations, and connecting students' learning experiences with real-world contexts (Pittalis et al., 2025; Wei et al., 2025). These approaches to teaching functional thinking should also be incorporated into pre-service teachers' pedagogical content knowledge. Thus, technological support becomes important for enhancing mathematics learning more broadly.

CONCLUSION

Fundamental Finding: The Indonesian elementary mathematics curriculum documents from Phase A to Phase C do not adequately represent functional thinking, despite demonstrating strength in recursive patterning. Although implicit potential for both aspects of functional thinking—covariational and correspondence—appears in some

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sections of the documents, their presence lacks coherence. The significant difficulties observed among students during the transition to formal algebra can be associated with the absence of functional thinking. Moreover, pre-service mathematics teachers' knowledge for teaching functional thinking was also found to be weak. **Implication:** The results across the three dimensions in this study suggest the need for adjustments to the algebra component of the primary mathematics curriculum, with greater emphasis on structured, gradual, and interconnected development of functional thinking. The growing pattern in Phase B should not only focus on a single pattern, but also include an approach that emphasizes the relationship between two simultaneously changing number patterns and guides students to articulate the relationship between those changes. In Phase C, the initial foundations of covariational reasoning in the topic of ratio and proportion should be strengthened by encouraging students to articulate the relationship between two quantities. Additionally, elementary teacher education programs must prepare teachers with the content and pedagogical knowledge of covariational reasoning as the foundation for understanding the meaning of variables and functional relationships. Functional thinking, as a whole, should be included in teacher education programs to better support students transitioning from pattern recognition to formal algebra. **Limitation:** This study has several limitations, including the use of a qualitative approach, which restricts the generalizability of its findings, and the selection of student and pre-service teacher samples, which may not proportionally represent the broader population. Potential researcher bias in selecting the study participants should also be considered. Moreover, curriculum analysis based solely on documents remains limited, and actual classroom implementation may not align with the formal documentation. **Future Research:** Given the multidisciplinary nature of elementary teacher education programs in Indonesia, mathematical content may not be studied in sufficient depth. Therefore, further research is needed on pre-service teachers' understanding of early algebra—particularly functional thinking—as well as on various interventions that may support their development.

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