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Mathematical Processes and Modeling in Problem-Solving: Designing Learning Models for Higher-Order Thinking

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ABSTRACT
21st-century mathematics learning requires the development of Higher Order Thinking Skills (HOTS) through meaningful problem-solving and mathematical modeling. Various studies have shown that students' HOTS abilities are still in the low category, even though problem-solving models have been widely implemented. However, most existing models are still procedural and have not systematically integrated horizontal and vertical mathematization processes with mathematical modeling cycles in a single learning syntax. This study aims to develop a problem-solving learning model grounded in mathematical processes and modeling to improve students' HOTS skills. The study uses the Research and Development (R&D) approach proposed by Thiagarajan and Semmel. The developed model integrates the stages of situational orientation, horizontal mathematization, vertical mathematization, contextual interpretation, and reflection and revision in a systematic and measurable learning syntax. The validation results from nine experts showed an average score of 3.61 in the very valid category, indicating that the model meets pedagogical feasibility, mathematical substance, and evaluative criteria. The novelty of this research lies in integrating Polya's steps, the mathematization process, and the mathematical modeling cycle into a learning model explicitly designed to develop HOTS. Further research is recommended to test the model's effectiveness through experimental studies in various educational contexts to strengthen empirical evidence of its implementation.

INTRODUCTION

Mathematical problem solving is broadly conceptualized as a higher-order cognitive activity that occurs when individuals face non-routine tasks without readily available solution schemes. From the perspective of cognitive theory and constructivism, problem solving does not simply involve algorithmic procedures but also involves structured reasoning, strategic decision-making, representational competence, and metacognitive regulation (Baumanns & Rott, 2022; Chuderski et al., 2021; Hamzah et al., 2022; Rott et al., 2021). In line with the demands of 21st-century competencies, problem-solving-based learning is expected to develop Higher Order Thinking Skills (HOTS), particularly in analysis, evaluation, and creation. Therefore, problem-solving has become a key pillar in contemporary mathematics curricula, including at the higher education level.

The preliminary analysis of this study indicates that university students' problem-solving abilities have not yet developed optimally, particularly at the HOTS level. (Purnomo et al., 2022, 2024; Sulistyaningsih et al., 2021). Most students fall into the low- and medium-ability categories and have difficulty applying mathematical concepts to complex contextual problems. Ongoing learning tends to be oriented toward procedural solutions and formulaic reproduction, thus not systematically facilitating analysis, evaluation, and creation. Furthermore, the reflection and solution validation stages are rarely conducted in depth, resulting in students being less accustomed to critical reasoning and strategy revision. This situation indicates a gap between the demands of 21st-century mathematics learning, which emphasizes contextual problem-solving and higher-order thinking skills, and university learning practices, which have not yet fully integrated mathematization and modeling in a structured manner. (Schukajlow et al., 2023; Zhu et al., 2025) .

However, recent research indicates a conceptual and pedagogical gap. Although the literature consistently emphasizes the urgency of HOTS-oriented learning (Thao, 2025), Many problem-solving models remain solely procedural and strategic, without explicitly integrating the cognitive transformation from real-world contexts to formal mathematical structures. Empirical evidence shows that students still make comprehension, transformation, and process-skill errors when completing HOTS tasks. (Fadzil et al., 2025). These findings indicate that the problem lies not only in conceptual mastery but also in limitations in constructing coherent mathematical representations and models.

Theoretically, these limitations stem from the lack of systematic integration of mathematization processes into a problem-solving framework. From a Realistic Mathematics Education (RME) perspective, horizontal mathematization enables students to transform contextual phenomena into informal mathematical representations (Agusdianita et al., 2021; Anggraini & Fauzan, 2020; Wardono et al., 2016). In contrast, vertical mathematization supports formalization, abstraction, and symbolic generalization. However, previous research tends to discuss mathematization and mathematical modeling separately, or to focus on testing the effectiveness of specific strategies without formulating an integrative pedagogical design explicitly aligned with HOTS indicators.

Based on these conceptual and pedagogical gaps, this study aims to develop and validate a problem-solving learning model based on mathematization and mathematical modeling processes through a Research and Development (R&D) approach. Specifically, this study designs an operational learning syntax that integrates problem-solving heuristics, horizontal and vertical mathematization, and the mathematical modeling cycle into a coherent didactic structure. This model is constructed to facilitate the mathematization trajectory from contextual phenomena to informal mathematical representations, continuing to symbolic formalization, generalization, and abstraction, and returning to the context through an iterative process of interpretation, validation, and model revision. Thus, this model explicitly structures students' mathematical activities within a framework of representational and operational transformation that demands coordination between analytical, evaluative, and constructive reasoning as the core of Higher Order Thinking Skills (HOTS). To achieve these objectives, this study is formulated in the following research questions: 1). What is the level of validity of the problem-solving model design based on mathematization and mathematical modeling processes developed? 2) What are the characteristics of the conceptual structure of a model that integrates horizontal mathematization, vertical mathematization, and the mathematical modeling cycle in a coherent syntax? 3) What is the potential of the developed model design in facilitating the development of students' Higher Order Thinking Skills (HOTS)?

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Theoretically, this study contributes to the mathematics education literature by formulating an instructional design framework that integrates the process of mathematization as an epistemic mechanism and mathematical modeling as an applicative praxis in a single, unified learning system. Unlike previous studies that tend to position problem solving, mathematization, and modeling as separate domains, this study offers an operational formulation that emphasizes the functional relationship between the three in building mathematical knowledge structures from the level of informal representation to axiomatic formalization. This contribution enriches the global discourse by providing an explicit learning trajectory in developing higher-order mathematical thinking skills that are not only procedural but also reflective, generative, and oriented towards the construction of meaning.

RESEARCH METHOD

This research is a research and development, namely the development of a problem-solving model based on mathematization processes and mathematical modeling to improve students' HOTS abilities. The development model used is a modified 4-D model from Thiagarajan, Semmel, and Semmel. The Thiagarajan, Semmel, and Semmel development model consists of 4 stages: define, design, develop, and disseminate. This study consists of only 3 stages. Problem-solving questions with the HOTS category with the abilities of Analyzing, Evaluating, and Creating (Anderson et al., 2001; Wilson, 2016).The data collection technique uses triangulation, namely conducting tests, observations, and in-depth interviews. (Creswell, 2014; Sukestiyarno, 2020). Data analysis uses an inductive approach, in which conclusions are drawn from detailed investigations of small cases to provide a big picture.

This research is a research and development, namely the development of a problem-solving model based on mathematization processes and mathematical modeling to improve students' HOTS abilities. The development model used is a modified 4-D model from Thiagarajan, Semmel, and Semmel. The Thiagarajan, Semmel, and Semmel development model consists of 4 stages, namely: define, design, develop, and disseminate. In this study, the dissemination stage has not been carried out. This limitation is based on the focus of the research, which is still in the initial development phase, emphasizing the construction and validation of prototype learning models conceptually and structurally.

The dissemination phase of the 4-D model is primarily concerned with broad implementation, cross-context testing, and adoption by practitioners. Because this study does not yet include empirical effectiveness testing through widespread field implementation, the dissemination phase has not been conducted. Therefore, this limitation is methodological and consistent with the principle of phased development in research and development. The dissemination phase is planned as part of further research that will involve implementing the model in real-world contexts, testing its effectiveness through experimental or quasi-experimental designs, and disseminating it to the practitioner and academic community. This study uses HOTS problem-solving with the capabilities of Analyzing, Evaluating, and Creating (Anderson et al., 2001; Wilson, 2016). Data collection techniques include triangulation, which involves conducting tests, observations, and in-depth interviews (Creswell, 2014; Sukestiyarno, 2020). Data analysis uses an inductive approach where conclusions are drawn from

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detailed investigations of small cases to provide a big picture. The research stages can be seen in the summary below.

1. Define Stage: Needs Analysis

This stage aims to identify the gap between students' actual HOTS abilities and the demands of context-based problem-solving learning. Data analysis consists of data reduction, data presentation, and conclusion. The needs analysis was conducted through 1) HOTS diagnostic tests compiled based on the revised Bloom's Taxonomy (Anderson & Krathwohl), 2) semi-structured interviews with students, and 3) learning observations to identify ongoing problem-solving practices. Data analysis used a qualitative approach, combining reduction and coding techniques with both deductive and inductive methods. The coding framework was compiled based on HOTS literature and problem-solving errors, including:

a. Comprehension Error

Inability to identify relevant information or understand the context of the problem (indicator: error in the analysis stage).

b. Transformation Error

Inability to transform a contextual problem into a mathematical representation (indicator: failure in the horizontal mathematization process).

c. Process Skill Error

Errors in symbolic or procedural manipulation (indicator: weakness in vertical mathematization).

d. Evaluation/Reflection Gap

Lack of validation or reflection on the solution (indicator: weakness in the evaluation and creation stages).

The next step is to verify the data with in-depth interviews. The interviews use a semi-structured protocol with three main domains: Cognitive Domain, with a grid of questions: 1) How students understand contextual problems; 2) What difficulties arise when converting context into mathematical form. Strategy and Representation Domain, with a grid of questions: 1) What strategies are used to solve problems? 2) What strategies are used to solve problems? Did students use diagrams, tables, or other models? Reflection and Evaluation Domain, with a grid of questions: 1) Did students review solutions? 2) How did the lecturer facilitate reflection? Interview data were transcribed and analyzed to identify patterns of cognitive gaps, which then informed model design. The final step was to complete the field data collection. Data collection techniques used were interviews, questionnaires, and observation.

2. Design Stage

At this stage, a learning syntax was designed that integrates Polya's problem-solving heuristics, horizontal and vertical mathematization processes, and the mathematical modeling cycle (construction, operation, interpretation, validation, and revision) in a coherent flow from the real context to mathematical abstraction and back to the context for reflection. Each stage is formulated in student activity indicators that emphasize analysis, evaluation, and creation as the core of HOTS, with the lecturer acting as a facilitator through scaffolding and questions that trigger higher-order thinking. The evaluation tool is structured in the form of an analytical rubric that measures the

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quality of the model's representation, procedural accuracy, argumentation, and reflection. Experts then validated the design to ensure theoretical suitability, substantive accuracy, and instrument measurability before implementation.

3. Develop Stage

The model was validated by experts, including learning model experts, mathematicians, and education evaluation experts. Validation aspects include: syntax coherence and theoretical consistency; the accuracy of mathematical concepts and the mathematization process; the integration of mathematical modeling; and the suitability of evaluation instruments for HOTS indicators. Revisions were carried out iteratively through the following steps: quantitative analysis of validation scores; classification of qualitative comments into syntax, concept, and evaluation aspects; revision of model components with the lowest scores; and re-checking consistency.

RESULTS AND DISCUSSION

Results

The research began with an analysis of the need for a problem-solving model. The results of the lecture evaluation and the study of needs for developing problem-solving models showed that students assessed several key aspects related to the ability to use Higher Order Thinking Skills (HOTS) and to build learning models, with varying scores. The evaluation results are shown in the image below.

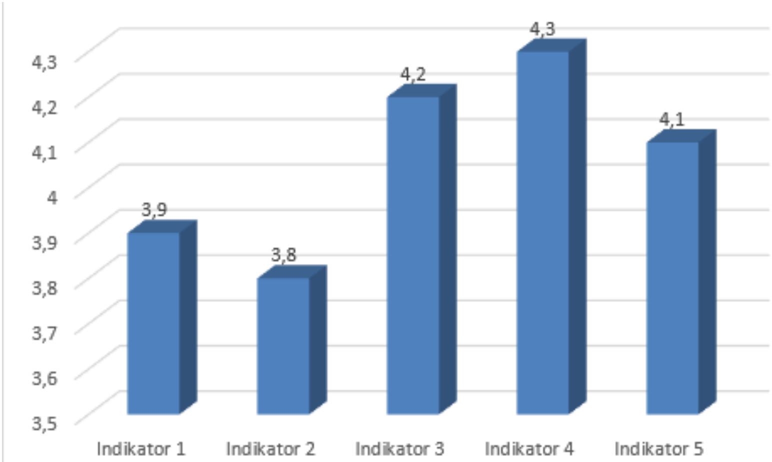

Figure 1. Lecture evaluation results

Based on Figure 1, the study's results can be summarized as follows: Indicator 1: Evaluation of the implementation of lectures received an average score of 3.9. This indicates that, in general, lecture implementation is considered quite good, but there is still room for improvement, especially in supporting the development of high-level problem-solving skills. Indicator 2: Analysis of the use of HOTS ability obtained the lowest score, 3.8, indicating that the implementation of HOTS in the learning process was not optimal. Students feel that they are not fully involved in activities that require analytical, creative, and evaluative thinking. The need for a problem-solving model scored 4.2, indicating that students urgently need a learning model specifically designed to help them master problem-solving, especially those related to HOTS questions.

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Indicator 3: The analysis of improving HOTS's problem-solving ability received the highest score, 4.3, indicating a high urgency to enhance this ability. Students realize that these skills are essential for academic success and preparation for the challenges of the world of work. Indicator 4: The target of increasing HOTS received a score of 4.1, indicating that students have high expectations for improving high-level thinking skills through the right learning strategy. The second evaluation focused on students' HOTS abilities. The results of students' HOTS abilities can be seen in the image below.

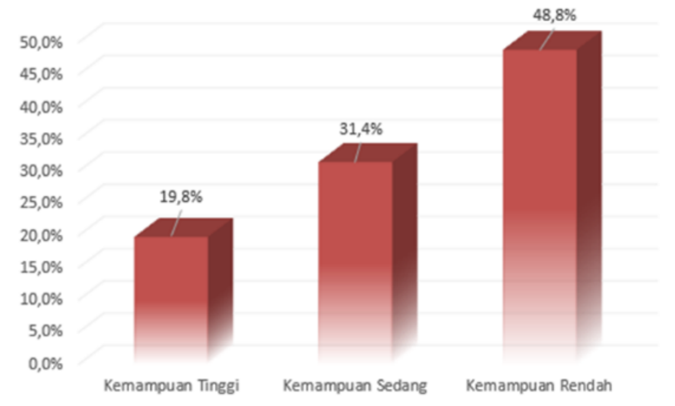

Figure 2. Student HOTS ability results

Based on Figure 2, which presents the research results in a bar diagram, students' problem-solving ability is distributed across three categories: high, medium, and low. The percentages of students in the high, medium, and low ability categories are 19.8%, 31.4%, and 48.8%, respectively. Overall, these findings indicate that the majority of students, which is 80.2%, are still in the low and medium ability category, so strategic efforts are needed in learning to improve high-level thinking skills, such as through the application of problem-based learning, project-based learning, and providing training that emphasizes contextual and open problem-solving.

The next stage is design. At this stage, the Polya problem-solving model serves as the primary reference in mathematics learning because it provides a systematic set of steps from understanding the problem to re-examining the solution. The stages of the problem-solving model according to Polya include: (1) understanding the problem, (2) planning a solution strategy, (3) implementing the plan, and (4) re-examining the results. However, this model remains general and needs further development to better align with the analysis of the needs and demands of contextual and complex problem-solving in the current era. One way to complete the Polya solution is to include the mathematicization process. The process of mathematicization is the initial stage in which students are taught to convert real situations or problems into mathematical forms. At this stage, students are expected to identify key elements in the problem and use mathematical representations to model the situation. (Hadi et al., 2018) stated that complex problems often result in difficulties for students, which can be overcome by presenting relevant and interesting situations to improve understanding of the problem. Thus, students are expected to build a good sense of how to formulate problems in a more formal mathematical context.

After the mathematicalization process, the next stage is mathematical modeling, in which students apply the models they have created to find solutions. In this context, Krulik and Rudnick's models can provide structure for problem-solving approaches.

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This model explains that problem-solving involves several steps: understanding the problem, planning the solution, executing the plan, and reviewing the results. By utilizing this process, students not only learn to find solutions but are also trained to think critically and creatively in evaluating the approaches and strategies they have implemented. Problem-solving models, mathematical processes, and mathematical modeling are conceptually and operationally connected. Problem-solving provides a framework, mathematics offers a thinking mechanism, and mathematical modeling provides the most comprehensive approach to solving contextual problems. The three form a unit that can significantly develop high-level thinking skills. The following draft troubleshooting model is shown in the table below.

Table 1. Troubleshooting Stages Design Table

Field	Mathematics	Mathematical Modeling	Objectives in HOTS Questions	Design of Problem-Solving Models
Understand the problem	Situational orientation	Understanding the real-world context	Determine what is being asked and what data is available.	Understanding the problem & identifying information
Motto to plan	Horizontal mathematicization	Simplify & make assumptions	Converting contextual problems to initial mathematical representations.	Create an initial representation.
Execute the plan	Vertical mathematicization	Operating mathematical models	Completing calculations, choosing strategies, and utilizing advanced concepts.	Processing mathematical models
Look back	Back to context	Interpretation & validation	Check whether the results are contextual, logical, and qualified.	Interpret & validate results.
Look back	–	Model revision	Fix the steps if there are any inconsistencies.	Reflection & revision

Based on Table 1, the developed model shows a systematic integration among the problem-solving stages, the mathematization processes (horizontal and vertical), and the mathematical modeling cycle, explicitly linked to the development of HOTS. Each stage forms a coherent cognitive flow, starting with situational orientation and context analysis (analyzing), followed by the construction and operation of the model through horizontal and vertical mathematization (creating), and culminating in its interpretation, validation, and revision (evaluating and metacognitive reflection). This structure shows that HOTS development does not occur implicitly. However, it is operationally designed in the learning syntax, so that students not only solve problems procedurally, but also build, evaluate, and revise models systematically and reflectively. Based on the analysis results, the following integrations can be made.

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Table 2. Integration of Problem-Solving, Mathematization, and Mathematical Modeling Stages with HOTS-Oriented Indicators and Student Activities

Integration Stage	General Indicators	Student Activities
Understanding Problems & Identifying Information (Polya: Understand, Mathematics: Orientation, Modelling: Context identification)	1. Mention the problem again in your own language. 2. Identify the objectives/questions that need to be answered. 3. Determine the available and needed data. 4. Identify relationships between information.	1. Read the questions carefully and then retell them in their own words. 2. Mark important information. 3. Mention the initial assumptions.
Making Initial Representations (Polya: Devise a plan, Mathematization: Horizontal, Modelling: Assumptions & Simplifications)	1. Changing contextual problems to the initial model (tables, diagrams, equations). .2. Making assumptions or simplifying problems. 3. Choosing a solution/strategy.	1. Sketch the situation. 2. Determine variables and parameters. 3. Write down the initial mathematical relationships.
Processing Mathematical Models (Polya: Carry out, Mathematics: Vertical, Modelling: Model operations)	1. Use relevant concepts/formulas. 2. Perform calculations with the correct procedures. 3. Arrange the solution steps logically.	1. Solve the equations that have been made. 2. Use advanced mathematical operations. 3. Check the consistency of calculations at each step.
Interpreting & Validating Results (Polya: Look back, Mathematics: Back to context, Modelling: Validate results)	1. Converting mathematical results back to the context language. 2. Evaluate the fairness of the results. 3. Check the fit of the results with the original purpose.	1. Explain the meaning of the answer in the context of the question. 2. Check whether the results make sense logically and realistically.
Reflection & Revision(Polya: Advanced Look back, Modelling: Model revision)	1. Identify the weaknesses of the solution. 2. Develop alternative strategies/models. 3. Correct calculations or assumptions if needed.	1. Change the initial assumptions if the results are not suitable. 2. Try other approaches to solve the problem.

The table shows the systematic integration between Polya's problem-solving stages, the mathematization process (horizontal and vertical), and the mathematical modeling cycle operationalized through indicators and student activities. Each stage forms a progressive cognitive flow for developing HOTS, starting with context analysis and information identification (analyzing), followed by the preparation of initial representations through horizontal mathematization, then formal processing and manipulation of models through vertical mathematization (creating), to interpretation, validation, and reflection to evaluate and revise solutions (evaluating and metacognitive). Thus, this table shows that HOTS development is explicitly designed and structured in the learning syntax through clear links between general indicators and student activities.

Develop

At this stage, the validation process is carried out to ensure that the developed learning model meets the theoretical, structural, and substantive aspects, making it feasible to test at the implementation stage. Validation involves three groups of experts, namely: (1) learning model experts, (2) mathematicians, and (3) educational evaluation experts. Each group consists of three validators who provide assessments based on relevant

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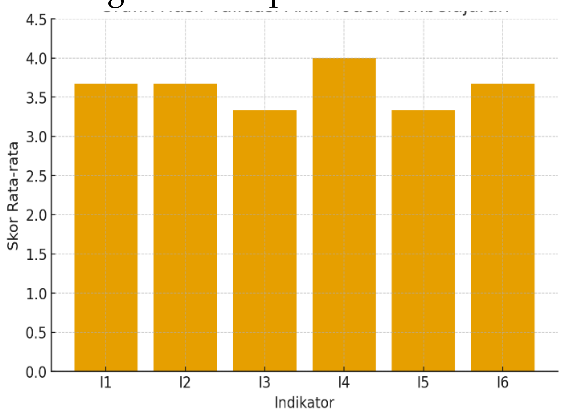
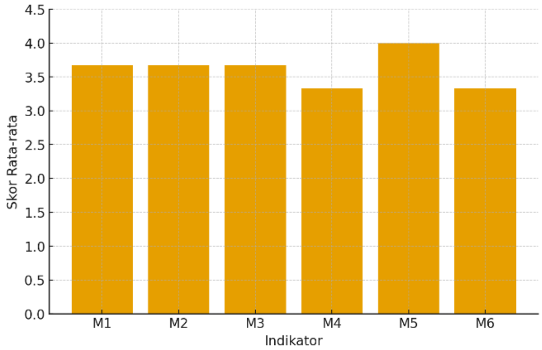
indicators according to their areas of expertise. The results of the analysis by the Learning Model Expert are as follows.



Indicator Description:
I1: Fit the model's objectives with the expected competencies
I2: Clarity of learning stages and syntax
I3: Suitability of teacher and student roles in the model
I4: Integration of model components
I5: Application of the model in the context of learning
I6: Model consistency with relevant learning theories

Figure 3. Expert Validation Analysis of Learning Model

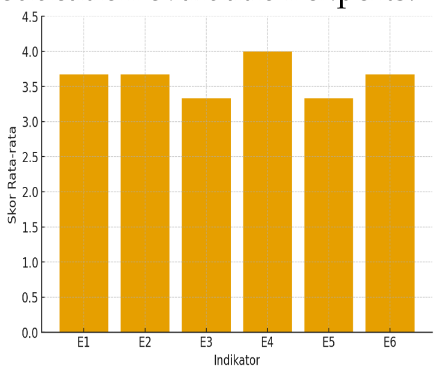
Based on Figure 3, the assessment results showed an overall average score of 3.61. Based on the validity criteria, the value falls into the "Very Valid" category. These findings show that the problem-solving model based on mathematics and mathematical modeling is perceived as having a clear structure, a coherent syntax, and well-integrated components, making it feasible from a pedagogical perspective. Next is the mathematician's validation.



Indicator Description:
M1: Truth of mathematical concepts
M2: Accuracy of the mathematical process
M3: Suitability of mathematical modeling elements
M4: Mathematical logic and argumentation rigor
M5: Clarity of mathematical representations
M6: Relevance of the material to the principle of HOTS capability

Figure 4. Mathematician Validation Analysis

Based on Figure 4, the total average was 3.61, which falls within the "Very Valid" category. The learning model developed is considered to have met the mathematical and scientific criteria. The model is considered able to accommodate horizontal and vertical mathematical processes appropriately and to facilitate mathematical modeling activities in accordance with academic and procedural rules. Next is the validation of education evaluation experts.



Indicator Description:
E1: Clarity of model objectives and alignment with evaluation competencies
E2: Accuracy of learning success indicators
E3: Quality of the assessment instrument compiled
E4: Measurability and scalability of the assessment rubric
E5: Consistency of assessment with the developed learning model
E6: The significance of the evaluation results for the development of HOTS

Figure 5. Analysis of Educational Evaluation Validation

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Based on Figure 5, the average was 3.61, with the "Very Valid" category. These results indicate that the evaluation instruments and assessment tools that accompany the learning model have been well prepared, measurable, and able to provide an authentic picture of students' development of abilities, especially in higher-level thinking. Overall, validation by the three expert groups provided very consistent results. The average score per validator group was 3.61, placing it in the "Very Valid" category. Thus, it can be concluded that the problem-solving model based on the mathematical and modeling process is declared feasible, theoretically valid, and ready to proceed to the field trial stage.

Discussion

The results of the study indicate that the developed problem-solving model based on mathematization and mathematical modeling processes has strong relevance to improving students' Higher Order Thinking Skills (HOTS). This is reflected in the assessment of three expert groups, which gave an average score of 3.61, categorized as very valid. This validity not only demonstrates the structural feasibility of the model but also indicates that the developed syntax design is able to respond to cognitive gaps identified at the needs analysis stage, particularly related to comprehension errors and transformation errors. Specifically, comprehension errors characterized by students' inability to identify relevant information and understand the problem context are addressed through the situational orientation and problem identification stages in the model syntax. At this stage, students are directed to reconstruct the meaning of the problem through reflective reading activities, identifying variables, and elaborating on relationships between information in a real-world context. This process serves to strengthen initial representations and ensure that students have adequate conceptual understanding before entering the solution stage.

Meanwhile, transformation errors, the inability to transform contextual problems into mathematical representations, are addressed through the integration of horizontal mathematization into the learning model. This stage explicitly facilitates students in building initial mathematical models through the use of representations such as diagrams, tables, and simple equations, as well as the formulation of assumptions and problem simplifications. Thus, students not only translate context into symbols but also construct meaningful mathematical relationships as a basis for the formalization process in the vertical mathematization stage. With a syntactic structure that systematically guides the transition from understanding context to mathematical representation, this model serves as a didactic mechanism that bridges the gap between students' conceptual understanding and representational abilities, which was previously a major source of errors in HOTS-based problem solving.

Initial research findings indicate that the majority of students (80.2%) still fall within the low and medium HOTS categories, and HOTS implementation in learning scored the lowest compared to other indicators. This situation indicates a gap between the demands of a HOTS-oriented curriculum and learning practices that do not fully facilitate higher-order thinking processes systematically. Therefore, the development of this model is directly aimed at addressing these weaknesses, particularly in the analysis, evaluation, and creation aspects that have not been structured in previous learning.

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This model integrates problem-solving steps, horizontal and vertical mathematization, and the mathematical modeling cycle, thereby requiring students to engage in higher-level cognitive processes. In the orientation and problem understanding stage, students are trained to analyze information, identify key variables, and determine relationships among problem components. This activity aligns with the analyzing category in Bloom's revised taxonomy. Students' analytical skills can improve significantly when learning begins with the exploration of contextual problems that require representing and grouping information. (Gunawan & Muflihati, 2022; Kolar & Hodnik, 2021). In the horizontal mathematization stage, students engage in an initial abstraction process, transforming real-world situations into informal mathematical structures such as diagrams, tables, or verbal representations. Horizontal mathematization encourages students to connect intuitive thinking with mathematical concepts, thereby enhancing their critical thinking capacity. This research also found that the ability to identify patterns and structures is fundamental to the development of HOTS, particularly in the analysis and evaluation aspects.

In the vertical mathematization and model formulation stage, students are required to develop formal models, use mathematical procedures, perform algebraic manipulations, and generalize patterns. This process involves creating within an HOTS framework. Research by Mathematical modeling activities can improve students' ability to design models, modify strategies, and engage in advanced reasoning. (Nur et al., 2021; Nurrosyadah et al., 2025; Purnomo et al., 2024). The model completion and interpretation stages train students to evaluate and reflect on concepts, which are core HOTS skills. When students evaluate mathematical results in the context of the problem, they validate the logic, assess the reasonableness of the solution, and review their initial assumptions. This activity aligns with Novitasari et al., (2020)research, which emphasizes that modeling-based learning can improve students' evaluative skills because they must ensure consistency between mathematical results and reality.

Furthermore, the validation and reflection stages in this model enable students to develop metacognitive skills, namely the ability to monitor and evaluate their own thought processes. Metacognition is one of the strongest predictors of HOTS development, as confirmed by Purnomo et al. (2022), who found that students engaged in problem-solving-based learning showed significant improvements in metacognitive and argumentative skills. The reflection phase in problem-solving and mathematical modeling is often a very important stage in the implementation of problem-solving (Baumanns & Rott, 2022; Purnomo et al., 2022, 2024; Rott et al., 2021)

The correlation between student needs and the model design is clearly evident in the developed syntax structure. The low level of HOTS skills in the initial stages was addressed by explicitly integrating mathematization processes and modeling cycles, which had not previously been fully supported in learning. A problem-solving-based learning model can effectively enhance HOTS when lecturers consistently integrate conceptual scaffolding and metacognitive questions (Kahar et al., 2025). Thus, the developed model not only meets the theoretical feasibility criteria but also demonstrates empirical fit with the learning problems identified during the needs analysis stage.

While this discussion has demonstrated strong theoretical support for the developed model, a more critical approach is needed, particularly in the Reflection & Revision stage. This stage requires students to conduct in-depth reflection, evaluate the strategies used, and revise the model based on validation results. In practice, this stage

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can face various challenges, such as limited learning time, a learning culture still oriented towards procedural solutions, and low levels of student reflection. Furthermore, lecturer preparedness is a crucial factor in the successful implementation of this model. Lecturers are not only required to understand the concepts of mathematization and modeling substantively, but also to possess the pedagogical competence to provide appropriate scaffolding, facilitate reflective discussions, and encourage critical student argumentation. Without such readiness, the reflection and revision phase risks not functioning optimally and becoming merely a formal activity lacking in deeper meaning.

These implementation challenges are directly related to the study's limitations, which did not fully explore contextual factors such as student characteristics, lecturer readiness, and learning environment conditions. Therefore, although the model is theoretically and structurally highly valid, its effectiveness in practice still depends on institutional support, lecturer training, and adequate time management for learning. This critical approach is essential for a more balanced and realistic discussion, highlighting not only the model's strengths but also considering the challenges of its implementation in the field. Further research based on design-based research or longitudinal experiments is needed to test the stability and sustainability of the model's impact on HOTS in various institutional contexts (Kahar et al., 2025; Purnomo et al., 2024). This critical approach makes the discussion more balanced, realistic, and reflective of the challenges of implementation in the field.

CONCLUSION

Based on the research results, it can be concluded that the problem-solving model based on mathematization and mathematical modeling processes is a valid, feasible, and relevant learning model for use in mathematics instruction oriented toward developing higher-order thinking skills (HOTS). First, validation results by three groups of learning model experts, mathematicians, and educational evaluation experts showed an average score of 3.61, categorized as highly valid. Second, this model has been proven to have a strong conceptual structure because it combines horizontal and vertical mathematization processes with the mathematical modeling cycle. Third, the developed model has great potential for developing HOTS, particularly the ability to analyze, evaluate, create, and reason mathematically.

These findings have important theoretical and practical implications. Theoretically, this research contributes to strengthening the framework of mathematics education by providing an operational formulation that integrates the mathematization process as an epistemic mechanism and mathematical modeling as an applied practice in a single integrated learning design. Practically, this model can be used as a reference for lecturers in designing more HOTS-oriented learning, particularly in developing students' analytical, evaluative, and creative thinking skills through contextual problem-solving activities. Furthermore, this model also has the potential to support the development of more reflective and metacognitive-based mathematics learning.

This study has several limitations. First, the research is still in the development and expert validation stage, so it has not yet empirically tested the model's effectiveness through direct classroom implementation. Second, the research context is limited to needs analysis and model design without incorporating a wider range of student characteristics and learning environments. Third, aspects of lecturer readiness and practical implementation factors have not been analyzed in depth, so potential obstacles to model implementation in the field have not been fully identified.

The main novelty of this study lies in the explicit integration of mathematical processes, modeling stages, and HOTS indicators into a single integrated cognitive pathway from contextual analysis to metacognitive reflection and model revision. Future research is recommended to test the model's effectiveness through experimental or quasi-experimental designs to measure its impact on significantly improving HOTS. Furthermore, follow-up studies should adopt a design-based research or longitudinal study approach to evaluate the sustainability of the model's implementation in various educational contexts. Future research is also recommended to examine lecturer readiness factors, scaffolding strategies, and the model's integration into the curriculum to ensure optimal and adaptive implementation to the needs of 21st-century mathematics learning.

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