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Layers of Student Understanding Based on Pirie Kieren's theory in Solving Story Problems in Terms of Cognitive Style

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ABSTRACT

Objective: Understanding lines and angles serves as the foundation for further mathematical topics such as trigonometry, geometry, calculus, etc. Students may struggle with advanced mathematical reasoning and problem-solving without a firm grasp of these basic concepts. This study aims to investigate how students' mathematical understanding processes are based on the folding back theory. **Method:** The research utilizes a qualitative approach with descriptive eksploratif design. Two subjects were selected from 28 seventh-grade students, each representing the field-dependent and field-independent cognitive style. Data was collected through mathematical comprehension tests, GEFT tests, and interviews. Data were analyzed through data reduction, data presentation, and verification stages, with each subject being interviewed to verify the processes. **Results:** Based on the research results, students in the field-independent cognitive style were much more active and better understood the problem-solving process than those in the field-dependent. However, both subjects still required learning assistance. **Novelty:** This research explores the folding back theory in the mathematical understanding process based on cognitive styles, whereas previous studies have mainly focused on mathematical comprehension abilities. Therefore, further research would benefit from using instructional media to better engage students in understanding the material.

INTRODUCTION

Education is an effort to prepare a golden generation through guidance, learning, and training activities to improve the quality of future generations, one of which is by taking formal education at school. In the learning process at school, there are various subjects, one of which is mathematics. This subject is familiar to the world of education, where mathematics is studied from elementary to tertiary education levels. Mathematics is a mandatory subject at every level of education because, in mathematics, students will think concretely, critically, logically, systematically, and interconnectedly between each piece of material. So, in studying mathematics, understanding skills are needed; this ability is essential in studying mathematics (Hikmah & Saputra, 2023).

According to Khalid et al. (2021), understanding is an essential ability for students to have because understanding means students' knowledge of concepts, procedures, and strategies for solving a given problem. So, the process of student understanding is an exciting topic to discuss because the process of Understanding is a process of growth and development of student thinking (Lynch et al., 2021). In understanding the process of growth and development of students' thinking, several theories have emerged that have discussed the process of student understanding; there are several theories, namely Skemp's in 1987, Hibert and Carpenter's theory in 1992, Piere-Kieren's theory in 1994, and many more researchers

who have to raise issues regarding the growth and development of students' understanding (Muliawati, 2020).

So, from the various theories that address the topic of understanding, the author is interested in Piere Kieren's theory, which considers that understanding is a growth process that is dynamic, perfect, and has several layers of intrinsic understanding (Mustikaningtyas & Susiswo, 2020). This never-ending growth process reveals that students go through several layers that are continuously repeated and never end; these layers of understanding can be illustrated like the layers of onion skin, wherein, for each layer of understanding, there are several different understandings or meanings in each layer. In the problem-solving process, folding back is necessary for students when they encounter difficulties, requiring them to return to a deeper level (George & Voutsina, 2023; Hähkiöniemi et al., 2023; Mangaroska et al., 2022; Marufi et al., 2022; Shabrina et al., 2023). There are eight types of layers of understanding, according to Pirie–Kieren (Asih et al., 2020; Gulkilik et al., 2020) where these layers include primitive knowing, image making, image having, Property noticing, formalizing, observing, structuring, and inventing. The allegations put forward by Pirie and Kieren, it is stated that the inventing layer often becomes primitive knowing, where the outermost layer of understanding always returns to the innermost layer of understanding, which is a feature of this theory, namely folding back or returning to the previous layer (Asih et al., 2020). Folding back in Piere Kieren's theory can occur at all layers of understanding. If a person cannot solve the problem completely, folding back or returning to the previous layer or understanding the material can occur. Figure 1 is an illustration of the layers of Piere Kieren's theory.

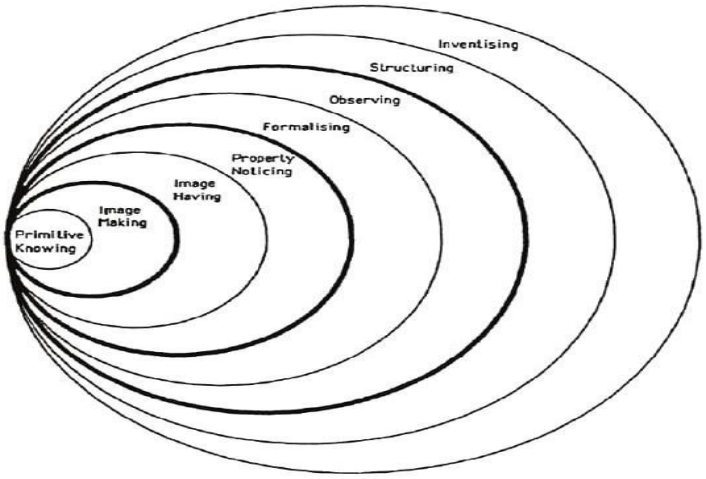

Figure 1. Layers of Piere-Kieren theory.

This image illustrates the layers of understanding in Piere Kieren's theory, where the image has information on each layer, the initial layer is primitive knowing, and the outermost layer is in invention. Then Susiswo et al. (2022) emphasized that Piere Kieren's theory can be divided into eight layers, where if a student can go to the outermost layer, the student's level of understanding will be higher, or if the student often experiences folding back and can solve problems up to the outermost layer. These students have high comprehension abilities. However, according to Hernama & Maharani (2023), the level of student success in studying mathematics is influenced by environmental factors, where the environment is very influential in the growth of students' mathematical understanding process.

Thus, the effectiveness of an understanding process influences the reception of information and habits. This is related to the learning environment; the learning process is an activity that is closely related to cognitive style (Purnomo et al., 2021). The cognitive style itself has the meaning of a characteristic found in each student and cannot be equated so that each student has a different cognitive style. It can also be called a student characteristic according to cognitive style. Students with the mindset to remember, solve problems, and make decisions without being influenced by others. In research conducted by Anggraeni et al. (2021), students' cognitive styles are not only characteristics but also references. Students can determine their own choices or answers. However, cognitive styles can be divided into two, with different characteristics and behaviors, where the styles are cognitive field-dependent and field-independent. The two characteristics of these cognitive styles will be the focus of research because these two cognitive styles are influenced by the conditions of the classroom environment, which will be the subject of research in the future. The field-dependent cognitive style is a student's character who is never separated from the help of other individuals; however, this is inversely proportional to the field-independent cognitive style. In this cognitive style, students are free and firm in their confidence level. Thus, researchers will focus on assessing the process of developing understanding that students have gone through based on Piere Kieren's theory as the focus of research and field-dependent and field-independent cognitive styles as a review in this research.

Many researchers have examined the process of mathematical understanding based on the folding back layers, referring to the Pirie-Kieren theory. Students fold back when solving limit problems and arithmogon problems in Pythagoras' theorem (Rahayuningsih et al., 2022). Layers of understanding of image making and image having for junior high school students with three-dimensional material. Amin & Sulaiman (2021), who examined folding back and the growth of mathematical understanding in Solving Mathematics Problems Based on Gender, and Ayuningtyas et al. (2024) described Elementary school students' layers of understanding in solving literacy problems based on the Sidoarjo context. Susiswo et al. (2023) analyzed the process of understanding function limit concepts in solving controversial problems based on Pirie Kieren's theory. However, no one has yet analyzed the mathematical understanding process based on the folding back layers regarding cognitive styles in lines and angles. According to the background of this research, the author aims to explain or describe how students' layers of understanding are based on Piere Kieren's theory of solving story problems in terms of cognitive style.

RESEARCH METHOD

The research method used in this research is a descriptive qualitative method. This method seeks to present quality findings in describing matters related to research. Furthermore, this method can find results that were by the researcher's objectives, namely, describing the folding back process that occurred at the level of students' understanding of Piere Kieren's theory in solving story problems in cognitive style. This research was conducted at one of the junior high schools in the western part of Indramayu district, namely Public Junior High School 1 Kroya. The procedures in this research can be seen in Figure 2.

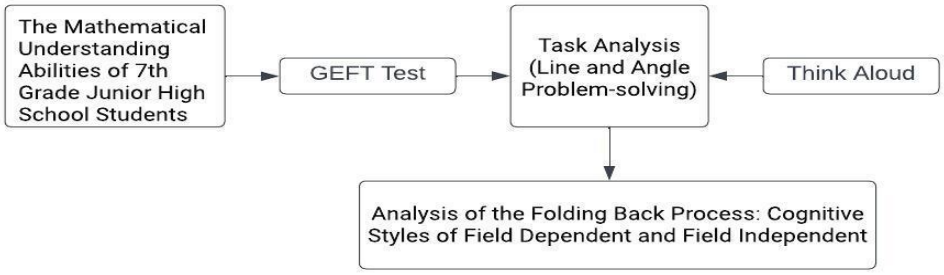

Figure 2. Research procedure.

Before conducting the research, the author validated the questions and interview guidelines used during the research. The author used expert validators, namely (1) two mathematics education lecturers and (2) two teachers who teach mathematics subjects from Public Junior High School 1 Kroya. These questions are created by the indicators the author uses, namely (1) restating a concept and planning a strategy/way to solve the problem. (2) changing the problem from the real world to mathematical form or vice versa and using symbols to complete arithmetic operations on the problem. After the validated instrument, during the research implementation process, the researcher took subjects from class VII-A, consisting of 28 students. The researcher will categorize subjects with low mathematical abilities into field-dependent and field-independent cognitive styles. To group subjects, researchers used written tests. The subject determination assessment uses mathematical question indicators to get an accurate subject. In the implementation, using a test which is a test to determine the grouping of subjects has been validated, namely the Geft Test, which Wiktin popularized; not only is the Geft test able to determine maximum results, but the researcher uses the results of interviews with selected subjects. The data analysis will be conducted in which the author uses overall data analysis techniques focusing on Miles & Huberman's opinion, including data reduction activities, data display, and conclusion drawing (verification).

RESULTS AND DISCUSSION

Results

In this research, the author used the Geft test, previously developed and popularized by Wiktin. The test has three stages, namely stage I, where students can work on seven questions for the practice process. After the practice process has been carried out, students immediately work on questions in parts II and III, which consist of 9 questions each. In the GEFT test questions, students are guided to look for a predetermined form; the process of working on the questions in each part is given a reasonably short time (around 15 minutes). Carrying out this Geft test does not need to require special skills and testing the validity of the questions because this Geft test has been tested, is valid and reliable, and has been tested many times; the process of taking subjects using the Geft test is intended to be able to determine subjects with students in the field dependent and field categories. Independent.

The GEFT test that the author has carried out can show that students fall into the field-dependent or field-independent category by looking at the results that the students have carried out. These students are used as references by researchers to be able to determine the results of the GEFT test that has been carried out, where students who get a score of 0-9 are classified as field dependent, and students who get a score of

10-18 are classified as field independent, in this research the subject is belonging to the cognitive style category can be seen through the following diagram:

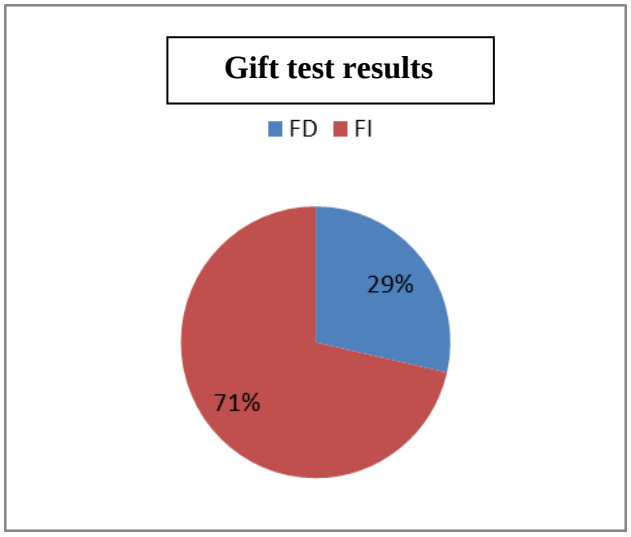

Figure 3. Categorization of cognitive styles.

Based on the representation in Figure 3, it is known that out of 28 students, it was found that 29%, around eight students, had a field-dependent cognitive style, and 71%, around 20 students, had a field-independent cognitive style. Next, to be able to determine the results of taking subjects after carrying out the student's mathematical test, which will be calculated from the results of their work on mathematics questions with scoring in the high category >8, medium 3 ≥ 8, and low <3, with this scoring, the author succeeded in determining the two subjects to be selected, where the subjects were students who had the low mathematical category S18 and S19. So that these two subjects are used as subjects by the author and have solved the mathematical problems as follows:

"Rian was camping, and he decided to look for firewood for cooking, but when he finished looking for firewood, he realized he was lost in the middle of the forest. To get to the camp, he used a compass. Rian's point is now at point A, then he walks east and stops at point B. After looking at the compass, he turned southwest at around 33°, arrived at the rocks at point g, then continued his journey to the southeast and met the crossroads at point h so that Rian continued his journey without stopping to the east and arrived at camp, the size between the gel points is 147°, determine the value of the high point through which the rain has passed (Figure 4)!"

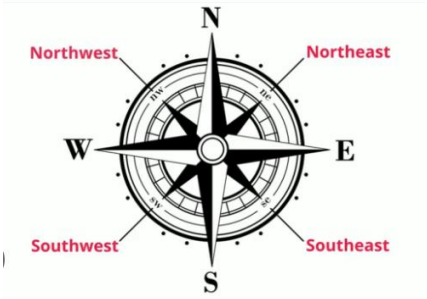

Figure 4. Cardinal direction.

Based on the results of the research that the author has carried out by the indicators that have been determined, where the two subjects have different cognitive style categories, they have succeeded in reaching the layer of mathematical understanding

where there are layers of primitive knowing, image making, image having, property notching, formalizing, observing, structuring. Moreover, finally inventing, the two subjects experienced different processes until they reached the inventing stage and experienced different folding back processes.

So, in the research process, the S18 FIR subject cannot illustrate the questions in pictures, but the subjects can write down what they know and are asked in the questions. With these instructions, the subject should be able to complete the initial steps in working on the questions. The subject confirmed that he could only work on the questions at an early stage, so he could not work on the questions ultimately. During the interview process, the researcher provided assistance instructions until the subject understood the work process to completion. The following are the work results on the questions carried out by subject S18, as in Figure 5.

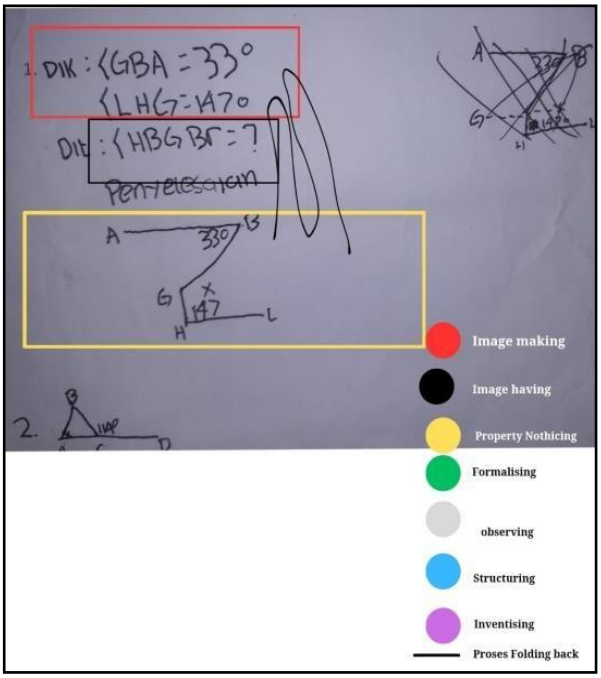

Figure 5. Results of answers from subject S18 FIR.

In Figure 5, the result of working on the questions on subject S18, where subject S18 was unable to complete the questions thoroughly, but when the author interviewed the subject, the subject gave the understanding that the subject still could not grasp what was meant by the process of working on the questions. Even though his teacher had previously provided the material. The following is an excerpt from an interview with the subject, which can conclude that.

Researcher (R): After that, what steps did you use to find <hgb?"
S18 : I do not know
R : If you ask you to understand the question again, in the question there are cardinal directions and additional clues, making it easier for you to find folding-back information.
S18 : yes
R : After you understand the question, can you illustrate the question in picture form?
S18 : Yes, there is another clue in the question that line x intersects point g
R : Once you understand these steps, what will you do? Explain
S18 : Draw it again, but it is the one with the x line; in the answer, I can do it

if you draw it, but it is finished; I do not know what else to do.

R : You have a semi-circular ruler; this ruler can help you determine the the angle of the folding back pk

S18 : That is what it is called, a bow ruler

R : Yes, did you know that angles and lines have a relationship?

S18 : Later, I feel like I have been taught the relationship between angles and lines

R : There are several kinds of angles and relationships between angles, can you remember them?

S18 : The one at the opposite angle, opposite, is that one-sided, right?

R : To be able to determine the next step, you must first understand what angles are in the problem

S18 : There are open one-sided angles that are <lhg, the same as the opposite angles.

R : What are the descriptions of one-sided angles and opposite angles? Explain

S18 : If opposite angles have the same value, ka, are there any opposite angles that are <gba equal <gbx if one side is 147.

Based on the interview results, it can be understood that to solve FIR subject questions, it is verified that there is an error in the answer sheet, where the subject is unable to illustrate the question in the form of a picture. However, the subject already knows what is known and asked in the question, which is <hgb, but during the interview, the subject needed help to answer how to begin searching for the results of <hgb. Therefore, the researcher provides additional understanding of line and angle materials to the subject experiencing the process of folding back from the layer of Property noticing to the layer of image making. Then, the subject can proceed to the image-making layer. However, when the subject reaches the image-making layer, they cannot answer the researcher's questions, necessitating them to fold back to the deepest layer of primitive knowing. The researcher provides additional understanding of line and angle materials, hoping the subject can grasp the information without straying from the topic. Next, the subject can proceed to the formalizing layer, where they understand the initial step to find <hgb by first finding <hgx. The subject has sufficient understanding of the material to explain that to determine the value of hgx; they need to illustrate again and identify opposite angles <gba, <gbx, and a unilateral angle <lhg. Consequently, the subject calculates 180 - <lhg = 180 - 147 = 33; thus, the value of <hgx is 33. The subject knows the next step is to find the angle <hgb after knowing the values of <bgx and <hgx by adding 33 + 33 = 66, resulting in <hgb being 66. After completing the problem, the subject is confident in their answers. However, they understand the problem-solving process better after being assisted by the researcher with additional understanding of the material. Figure 6 illustrates the folding back process that S18 has gone through.

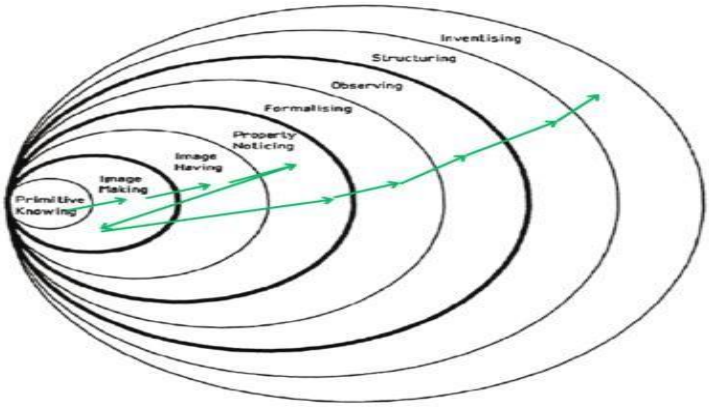

Figure 6. Folding Back S18 FIR.

Based on this picture, subject S18 completed the questions entirely with the teacher's help during the interview and experienced the folding back process twice. Then, subject S19 illustrated the question in the form of a picture quite well and could write down what he knew and was asked in the question. However, the subject needs help to solve the questions on the answer sheet; Figure 6 is the answer sheet for subject S19.

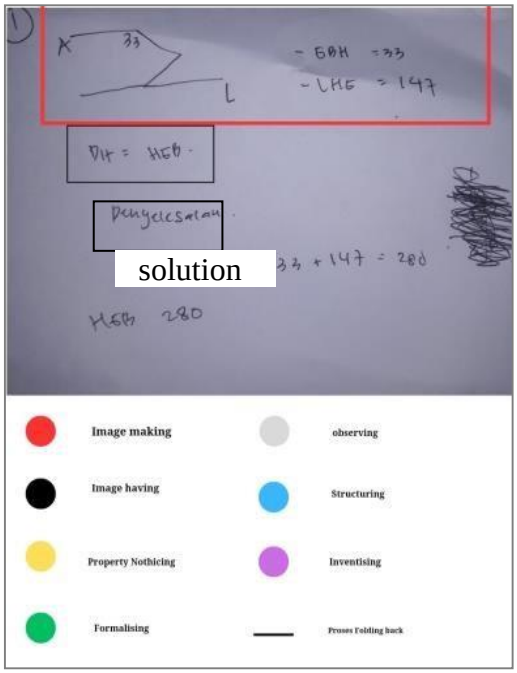

Figure 7. Results of answers from subject S19 FDR.

In Figure 7, subject S19 could not solve the problem completely, but when the subject was interviewed, he stated that he could not solve the problem entirely but tried to understand it.

Researcher (R): "After that, what steps did you use to find <hgb?"
S19 : I am confused
R : Please try to understand the question again
S19 : There is something that has not been drawn yet; there is a note in the question because line x intersects point g
R : What will you do next?
S19 : Should I redraw again?

R	*: After you redescribe it, how many kinds of angles do you know?*
S19	*: There are opposite corners, aren't you the one <gbd,<gbx so it's all 33, okay?*
R	*: What other angles do you know?*
S19	*: It's like an arc ruler, right? If you use an arc ruler, you can measure angles too. So, if the angle is <lhg, that means the remainder next to it is 33 ka, right? 147+33=180 ka, so it is like an arc ruler so you write the caption next to 147, which is 33*
R	*: What will you do next?*
S19	*: There is another opposite corner ka, the side 147 is equal to <hgx ka, meaning <hgx is also 33 ka*

Based on the interview quote, it states that subject S19 could not solve the questions on the answer sheet but could understand the basic concepts of the question material, and up to the property learning layer, the subject experienced problems where he did not understand the initial solution to get to the formalizing layer. Hence, the author provided learning assistance with parables. Students with a ruler and angle could determine the position of the angle contained in the problem. The subject folds back to the image-making layer and illustrates again so that he knows that there are opposite angles, namely <gba, <gbx, which have the value 33, and performs the addition so that to determine <hgb, we look for the value <hgx with 147+33=180. With this parable, the subject can proceed to the next layer with the help of the author until he reaches the inventing layer.

Next is an illustration of the process of understanding the subject of S19, which has been passed by going through the eight layers of Piere Kiern's theory and going through the folding back process once on the Property noticing layer to get to the formalizing layer, Figure 8 is an illustration of the folding back.

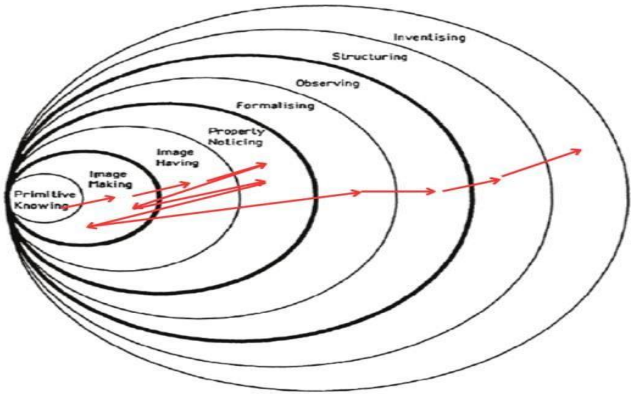

Figure 8. Folding Back from S19 FDR.

From the interviews with the two subjects, the author concludes that the folding back process carried out by each subject is different and has different thoughts for both subjects belonging to the field-dependent and field-independent cognitive style categories. Next, the author carried out data triangulation using the results from the FDR and FIR subjects. Based on the description and analysis of the FDR and FIR data, it can be concluded that the subject's folding back understanding layer in solving line and angle problems is in Table 1.

Table 1. Tables and figures should be valuable, relevant, and visually attractive.

No	Layer understanding folding back	S19 Field Dependent	S18 Field Independent
1	Primitive knowing	Have understanding Start by knowing the material that will be tested	Have an understanding beginning by knowing the material to be tested
	Conclusion	Subjects S1 and S7 have an understanding of early ones related to material line and corner	
2	Image making	Capable making description as a general stage of problem-solving	Capable making description as a general stage of problem-solving
	Conclusion	Subjects S1 and S7 are capable of making descriptions general with understand the problem of the question	
3	Image having	Able to know problems without doing examples	Able to know problems without working on examples
	Conclusion	Subjects S1 and S7 are capable of knowing problem questions without finish in a detailed way And do examples _	
4	Not hiding Property	Succeed connects with the picture problem without explaining it in detail.	Connecting successfully with illustration problems without explaining them in detail.
	Conclusion	Subjects S1 and S3 can connect a general overview of the planning concept and the solution question.	
5	Formalizing	Able to apply the problem-solving process that was known at the previous level.	Able to apply the process solution question identified with improvements so that two folding back events occur in the primitive knowing And image making.
	Conclusion	Subject S1 can apply the solution question without correction; Subject S7 experiences a two-time process of folding back with a return to layer primitive knowledge and image making so that the subject can be capable when he has gone through the folding back process.	
6	Observing	Capable of making observations from solution with repair so that happen folding back to the noticing property level.	Able to make observations from previous solutions.
	Conclusion	Subject S1 experiences a folding back process and returns to the primitive knowing layer so that the subject can be said to be capable when it has passed the process of folding back; Subject S7 is capable of doing observation from solution questions at the previous level.	
7	Structuring	compile solutions about material lines And corners, which have been given based on the completion process at the previous level.	Compose solutions to problems regarding lines and angles that have been presented based on process solutions on the previous level.
	Conclusion	Subjects S1 and S7 can do solutions from the previous stage until the stage determines the problem results.	
8	Inventing	Cannot create new questions	Unable to create an inquiry new

No	Layer understanding folding back	S19 Field Dependent	S18 Field Independent
		from already learned questions; however, can finish question with complete.	from the question that has been studied, but can be completed question with complete.
	Conclusion	Subjects S1 and S7 can complete a question, but No can make a statement new from question Which has done.	
9	Folding back	Return process to deepest layer levels understanding previously to solve problems without going off-topic once.	The process returned to the most profound layer, m-level understanding previously for solving problems without going off topic twice.
	Conclusion	• Subject S1, with style cognitive Fields dependent, can complete the question with the process Of one-time folding back from layers observing to layer formalizing. • Subject S7 with a field independent cognitive style can finish question with two time folding back from the formalizing layer to the nothicing property layer. However, the subject still cannot carry on to levels, so it folds back to return to layer image-making until it can return to levels.	

Table 1 states that FDR subjects with a field-dependent cognitive style can solve problems with a one-time folding back process from the notching property layer to the image-making layer. Moreover, FIR subjects with a field-independent cognitive style can solve the problem by folding back twice from the property mothering layer to the image-making layer. However, the subject still cannot proceed to the next level, so folding back returns to the primitive knowing layer until they can return to the next level. Then, the author illustrates the process of understanding the subject, passing through the eight layers of Piere Kiern's theory and going through the folding back process once on the property notching layer to get to the formalizing layer. Figure 9 is an illustration of the folding back process. Based on Figure 9, the two subjects have significant differences. The s18 FDT subject did two folding backs, and the FIT subject did one folding back.

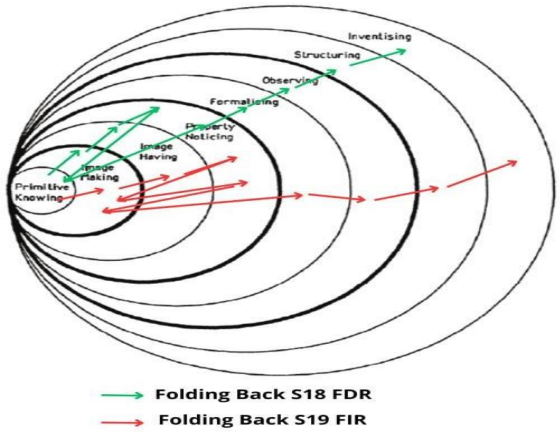

Figure 9. Results of answers from subject S18 FIR.

Discussion

FIR subjects are more interactive than FDR subjects, where FIR subjects should find it easier to interact with others, and FDT tends to be quieter and have difficulty expressing things (Mahfiroh et al., 2021). So, the research that the author has carried out found new results where subjects who have the field-dependent category are much more active and curious regarding solving the problems they have done. Subjects with the field independent category, but both subjects successfully solved the problem with the help of the author, where the author provides learning assistance. The teacher gives students time to complete the questions, but when students find it challenging to solve them, the teacher provides learning assistance. So that students are successful in continuing to solve the questions, a person needs time to think about working on the questions. The teacher provides learning assistance when the student cannot complete the task.

Then, for the folding back process that has been carried out by the two subjects where the FDT subject experiences two folding backs on the layer from notching properties back to image making and from image making steps back to the formalizing layer, but the subject is still unable to continue the completion because he feels it is lacking. Knowledge, so the subject decides to return to the image-making layer and reaches the formalization layer up to invention so that the subject of S18 FDT in the folding backflow is PK-IM-IH-PN-IM-PN-IM-F-O-S-I. for the results of the FIT subject experiencing one folding back which occurs at the noticing property layer back to image making until it can continue to the formalizing layer, both subjects carry out the folding back process in the same part, namely at the noticing property layer, for the folding back which s19 FIT has carried out in the folding back groove is PK-IM- IH-PN-IM-F-O-S-I. The more often someone folds back, the deeper the understanding of the material (Widyastuti et al., 2023), but in the research conducted by the author, the subjects did not understand the material they had completed to be able to solve the problem which could improve subject knowledge related to mathematical understanding, it is best to apply scaffolding to students. This happens because more than prior knowledge is required to solve the problem.

Scaffolding is assistance teachers give to students to solve problems in the learning process. Apart from that, scaffolding, in solving complex types of problems, optimizing mathematical understanding abilities is needed to increase the success of the learning process (DiNapoli & Miller, 2022; MacLeod & van der Veen, 2020; Sinha et al., 2021; Tegeh et al., 2021; Vogel et al., 2022; Zhong & Si, 2020). So, the writer should carry out the learning process to balance the different characters of students. Teachers carry out scaffolding actions with each student's attitude category, where students who have a field-dependent cognitive style category get scaffolding actions or assistance given to students can be in the form of pictures, instructions, motivation, and warnings, breaking down problems into solution steps, providing examples, and doing other things that allow students to learn on their own. This assistance is provided to help students solve mathematical problems without the help of others. In the scaffolding approach, assistance can be provided to both groups and individuals (Chen, 2020; Hou & Keng, 2020; Janson et al., 2019; Jarvis & Baloyi, 2020; Zhang et al., 2023). Assistance can be provided in unique places. So, learning assistance or scaffolding can be an alternative for teachers to improve students' mathematical understanding abilities (Fauzi & Chano, 2022; Ihechukwu, 2020; Ivars et al., 2020; Kim et al., 2022; Zhai, 2021).

CONCLUSION

Fundamental Finding: Based on the research results obtained by the author, it is stated that students who have the field-dependent cognitive style category understand the material of the questions that have been given by the author much better. This research does not align with previous research where subjects with the field-dependent cognitive style category are much more open and active. When conducting interviews with researchers, not with subjects in the field independent category who were much quieter and only talked a little. Next came the folding back process, where subject S18 FIR did two folding backs, and subject S19 FDR experienced one folding back. However, these two subjects still need to improve in working on the questions and using learning assistance from the author until they reach the final layer. **Implications:** This research aims to provide theoretical contributions related to the mathematical understanding process in solving angle and line problems, as viewed through students' cognitive styles, and to provide insights into the field of education, particularly in mathematics education. This research can be a reference for other researchers in examining student understanding issues. Teachers can provide exercises ranging from the simplest to the most complex problems. Additionally, students are expected to become more careful in reading, interpreting, or understanding the meaning of the problems to minimize errors in the problem-solving process. **Limitation:** Nevertheless, based on the results obtained, several weaknesses are still in mathematical understanding when considering the folding back theory. This study is limited to the cognitive styles of field-independent and field-dependent. **Future Research:** Subsequently, other researchers can explore the field-neutral cognitive style and develop scaffolding based on each cognitive style so that students' mathematical understanding abilities can be optimally achieved.

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